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# DYNAMICS OF A HIGH-REVOLUTION COMPRESSOR†

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The dynamics of a high-revolution compressor where each of the mountings is formed by two single-row ball bearings pressed into a common housing and considered. Springs with a rated force are set up between the housing and the body. Relations are obtained between the mass characteristics of the housings, the coefficients of rigidity of the elastic mountings and the frequency of rotation of the compressor for which the dynamic pressures on the mountings of an unbalanced rotating compressor vanish. Formulas are obtained which define the first two critical frequencies of rotation of a compressor in elastic mountings.

As THE frequency of rotation increases, the operating life of ball bearings when they are rigidly installed in the framework falls sharply since the pressure between the balls of a bearing and its external ring increases in proportion to the square of the angular velocity of rotation. According to the theory which is presented in courses in theoretical mechanics [1–3], in order the reduce the pressure on the mountings, it is necessary to reduce the static and instantaneous imbalance of the rotating solid to zero. A whole branch of technology, that is balancing technology, has been set up for this purpose. However, in practice, as a consequence of deformation, the reaction of ball bearings, starting from a rather low value of the eccentricity and angle which characterizes the instantaneous imbalance, continues to increase sharply at high values of the frequency of rotation, which also leads to the destruction of the bearings in spite of very careful balancing [4].

The installation of elastic mountings [5] between the external ring of a bearing and its housing became an alternative when designing efficient high-revolution machines mounted on ball bearings. However, their premature breakdown is observed when the rotor is installed in single-row ball bearings due to the misalignment of the cage with respect to the external ring of the bearing. It is shown below that, when mountings consisting of two single row ball-bearings pressed into a common housing which is mounted elastically in the body are used, all the advantages of a shaft in elastic mountings are preserved and there is no skewing of the cage.

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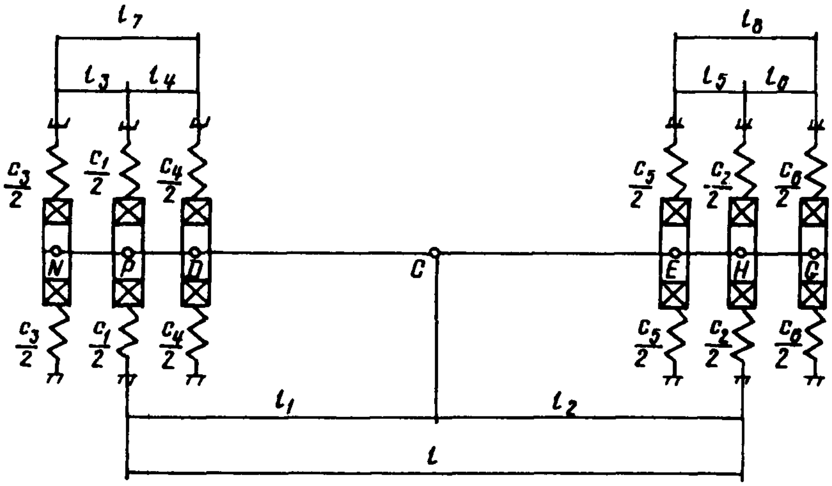


FIG. 1.

## 1. BASIC EQUATIONS

Let us consider an eight-stage compressor which is rotating in a pair of double elastic mountings. The double left-hand mounting is formed by two ball bearings which have been pressed into a common housing fixed to the framework by means of two rings with rigidities $c_3$ and $c_4$ (Fig. 1). The right-hand pair of mountings is formed in a similar manner and the rigidities of its springs are denoted by $c_5$ and $c_6$. We consider the compressor as a rigid horizontal shaft which is rotating in two elastic mountings with rigidities $c_1 = c_3 + c_4$ and $c_2 = c_5 + c_6$ with a centre of mass which is denoted by $C$. Equivalent elastic mountings are located at points $P$ and $H$. The equations for the small forced vibrations of such a shaft, which are caused by the static and instantaneous imbalance are

$$\begin{gathered} M(y_1^{\cdot\cdot}l_2 + y_2^{\cdot\cdot}l_1) + c_1y_1l + c_2y_2l = \mu\cos\omega t \\ M(z_1^{\cdot\cdot}l_2 + z_2^{\cdot\cdot}l_1) + c_1z_1l + c_2z_2l = \mu\sin\omega t \\ A\omega(y_2^{\cdot} - y_1^{\cdot}) - B(z_2^{\cdot\cdot} - z_1^{\cdot\cdot}) + c_1z_1l_1l - c_2z_2l_2l = -\nu\sin(\omega t - \varepsilon) \\ A\omega(z_2^{\cdot} - z_1^{\cdot}) + B(y_2^{\cdot\cdot} - y_1^{\cdot\cdot}) - c_1y_1l_1l + c_2y_2l_2l = \nu\cos(\omega t - \varepsilon) \\ \mu = Mel\omega^2, \quad \nu = (B - A)l\omega^2\delta \end{gathered} \tag{1.1}$$

Here, we have adopted the notation: $M$ is the mass of the compressor, $y_1$ and $z_1$ are the coordinates of an equivalent compressor mounting located at the point $P$ and the $x$ axis coincides at the equilibrium position with the axis of symmetry of the compressor, $y_2$ and $z_2$ are the coordinates of the second equivalent compressor mounting located at point $H$, $l_1$ and $l_2$ are the distances from the centre of mass to the mountings located at points $P$ and $H$, $l$ is the distance between the compressor mountings, $c_1$ and $c_2$ are the rigidities of the elastic mountings located at points $P$ and $H$, $A$ is the moment of inertia of the compressor with respect to the axis of symmetry, $B$ is the moment of inertia of the compressor with respect to any axis perpendicular to the axis of symmetry of the compressor and passing through the centre of mass, $\omega$ is the constant angular velocity of rotation of the compressor, $e$ is the eccentricity of the compressor, $\delta$ is the angle of deviation of the principal central axis of inertia from the geometrical axis of the compressor and $\varepsilon$ is the angle between the planes passing through the geometrical axis of the compressor and through the centre of mass and the angle $\delta$ respectively.

The first two equations of system (1.1) are the differential equations of the motion of the centre of mass. The second two equations of (1.1) are written in accordance with the theorem of moments regarding translationally moving axes, the origin of which coincides with the centre of mass.

Equations (1.1) hold when there is a small degree of imbalance of the rotor, that is, they must satisfy the relations

$$\delta \ll 1, \quad el^{-1} \ll 1, \quad (y_2 - y_1)l^{-1} \sim (z_2 - z_1)l^{-1} \sim \delta \sim el^{-1}$$

which are always satisfied when there is current balancing.

The particular solution of system (1.1), which determines the forced vibrations of the compressor, has the form:

$$y_1=A_1\cos(\omega t-\chi),\quad z_1=A_1\sin(\omega t-\chi)$$
$$y_2=A_2\cos(\omega t-\psi),\quad z_2=A_2\sin(\omega t-\psi) \qquad (1.2)$$

Here,

$$a_1=A_1\cos\chi=(\mu\xi_2-\nu\eta_{12}\cos\varepsilon)f^{-1},\quad b_1=A_1\sin\chi=(-\nu\eta_{12}\sin\varepsilon)f^{-1}$$
$$a_2=A_2\cos\psi=(\mu\xi_1+\nu\eta_{21}\cos\varepsilon)f^{-1},\quad b_2=A_2\sin\psi=(\nu\eta_{21}\sin\varepsilon)f^{-1} \qquad (1.3)$$
$$f=f(\omega)=-\xi_2\eta_{21}-\xi_1\eta_{12}$$
$$\xi_k=(B-A)\omega^2-c_kl_kl,\quad \eta_{ik}=Ml_i\omega^2-c_kl;\quad i,k=1,2$$

## 2. EQUATIONS OF MOTION OF THE HOUSING

Let us write the equations of motion of the housing into which the two left-hand mountings are pressed

$$m(u_3^{\cdot\cdot}d_2l_7^{-1}+u_4^{\cdot\cdot}d_1l_7^{-1})=-c_3u_3-c_4u_4+R_{u3}+R_{u4} \qquad (2.1)$$
$$I(u_4^{\cdot\cdot}-u_3^{\cdot\cdot})l_7^{-1}=-R_{u3}d_1+R_{u4}d_2+c_3u_3d_1-c_4u_4d_2;\quad u=y,z$$

Here, $m$ is the mass of the housing together with the non-rotating parts of the bearings, $I$ is the moment of inertia of the housing with the non-rotating parts of the bearings with respect to the horizontal axes $y$ and $z$ drawn through the centre of mass of the housing ($I=I_y=I_z$), $u_3$ are the coordinates of the compressor mounting located at point $N$, $u_4$ are the coordinates of the compressor mounting located at point $D$, $d_1$ and $d_2$ are the distances from the centre of mass to the mountings located at points $N$ and $D$, $l_7$ is the distance between the compressor mountings, $R_{u3}$ and $R_{u4}$ are the reactions on the housing as viewed from the spindle and $c_3$ and $c_4$ are the coefficients of rigidity of the elastic mountings located at points $N$ and $D$ (Fig. 2).

By putting

$$y_3=(l+l_3)l^{-1}A_1\cos(\omega t-\chi)-l_3l^{-1}A_2\cos(\omega t-\psi)$$
$$y_4=(l-l_4)l^{-1}A_1\cos(\omega t-\chi)+l_4l^{-1}A_2\cos(\omega t-\psi) \qquad (2.2)$$

($l_3$ and $l_4$ are the distances from the mountings located at the point $P$ to the mountings located at points $N$ and $D$) and solving the equations following from (2.1) jointly for the unknowns $R_{y3}$ and $R_{y4}$, we find that $R_{y3}=0$ and $R_{y4}=0$ for an arbitrary value of $t$ if the following equalities are satisfied:

$$J_1(l,\omega)=J_1(0,\omega)=0,\quad J_2(l,\omega)=J_2(0,\omega)=0$$
$$J_1(l,\omega)\equiv I\omega^2l_7-d_1d_2\omega^2m(l+l_3)+(c_4l_7^2-d_1^2\omega^2m)(l-l_4) \qquad (2.3)$$
$$J_2(l,\omega)\equiv -I\omega^2l_7+(c_3l_7^2-d_2^2\omega^2m)(l+l_3)-d_1d_2\omega^2m(l-l_4)$$

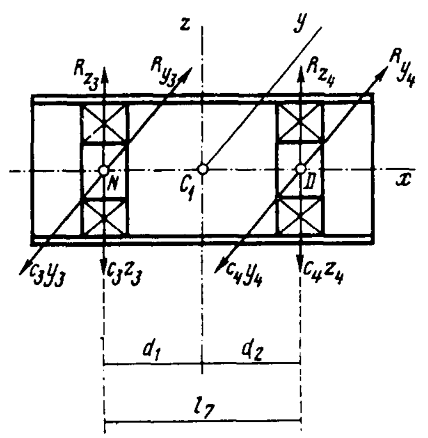


FIG. 2.

By simultaneously solving Eqs (2.3), we find three conditions and, when these conditions are observed, the two reactions between the compressor and the housing vanish when there is static and dynamic imbalance

$$c_3d_1=c_4d_2,\quad m=(c_3+c_4)\omega^{-2},\quad I=(c_3+c_4)d_1d_2\omega^{-2} \tag{2.4}$$

On treating the two right-hand mountings in the same manner, we arrive at similar conditions for the reactions between the compressor and the housing to be equal to zero when there is static and dynamic imbalance on replacing $c_3$, $c_4$, $d_1$ and $d_2$ in (2.4) by $c_5$, $c_6$, $d_3$ and $d_4$, respectively, where $d_3$ and $d_4$ are the distances from the centre of mass to the mountings located at points $E$ and $G$, and $c_5$ and $c_6$ are the rigidities of the elastic mountings located at points $E$ and $G$.

## 3. CRITICAL FREQUENCIES OF ROTATION

According to Eqs (1.3), the amplitudes of the forced vibrations caused by static and dynamic imbalance increase without limit when $f(\omega)=0$. By solving this biquadratic equation, we find that

$$\omega_{1,2}=[v\pm(v^2-2wc_1c_2l^2)^{1/2}]^{1/2}w^{-1/2}$$
$$v=(c_1+c_2)(B-A)+M(c_1l_1^2+c_2l_2^2),\quad w=2M(B-A) \tag{3.1}$$

The critical frequencies of rotation of the compressor in the two elastic mountings, when there are forced vibrations caused by the static and dynamic imbalance of the compressor, are determined by this formula.

## 4. SELF-CENTRING OF THE COMPRESSOR

Let us now consider the forced vibrations of the compressor when there is an unbounded increase in the frequency of rotation. From Eq. (1.3), we find the limiting values of the constants $a_1$, $b_1$, $a_2$ and $b_2$ when $\omega\to\infty$ and, then, the limiting values of the coordinates from Eqs (1.2)

$$\lim_{\omega\to\infty} y_k=-e\cos\omega t-(-1)^k l_k\delta\cos(\omega t-\varepsilon)$$
$$\lim_{\omega\to\infty} z_k=-e\sin\omega t-(-1)^k l_k\delta\sin(\omega t-\varepsilon);\quad k=1,2 \tag{4.1}$$

If the coordinates of the point of the geometric axis of the compressor lying at the intersection of this axis with a plane normal to the axis of rotation and passing through the centre of mass is denoted by $(y, z)$, then the coordinates of the centre of mass are

$$y_c=y+e\cos\omega t=(y_1l_2+y_2l_1)l^{-1}+e\cos\omega t$$
$$z_c=z+e\sin\omega t=(z_1l_2+z_2l_1)l^{-1}+e\sin\omega t \tag{4.2}$$

The angles which are formed by the principal central axis of inertia and the coordinates of the $xz$ and $xy$ planes are

$$\beta=(y_2-y_1)l^{-1}+\delta\cos(\omega t-\varepsilon),\quad \gamma=(z_2-z_1)l^{-1}+\delta\sin(\omega t-\varepsilon) \tag{4.3}$$

On substituting the resulting limiting values of the coordinates into Eqs (4.2) and (4.3), which determine the coordinates of the centre of mass of the compressor and the angle of deviation of the principal axis of inertia from the geometrical axis of the compressor, we find

$$\lim y_c=\lim z_c=\lim\beta=\lim\gamma=0 \quad \text{as} \quad \omega\to\infty$$

Hence, as the angular rate of rotation increases, the axis of rotation of the compressor tends to coincide with the principal central axis of inertia.

Consequently, when there is an unlimited increase in the angular velocity of the compressor, the static and dynamic imbalance of the compressor tends to zero, that is, the compressor is self-centring.

## 5. EXPERIMENT AND CONCLUSIONS

As experiment confirms, it is necessary to choose the rigidities of the equivalent elastic mountings $c_1$ and $c_2$ by determining the optimal value of the first and second critical frequencies of rotation. Experiment confirms that it is advisable to set the second critical frequency of rotation below the range of operational frequencies of rotation by 1000–2000 r.p.m.

In the compressor being considered and in a range of operation frequencies of rotation from 25000 to 45000 r.p.m., the rigidities were $C_1 = 0.319 \times 10^5$ N/cm and $c_2 = 1.07 \times 10^5$ N/cm. The calculated critical frequencies of rotation $n_1 = 11600$ r.p.m. and $n_2 = 23900$ r.p.m. are in good agreement with experimental data. In this case, the compressor was treated as a flexible body. The effect of self-centring, which has been proved when there is an unlimited increase in the frequency of rotation, developed at 1000–2000 r.p.m. after passing through the second critical frequency of rotation. For instance, at 25000 r.p.m., the amplitude of the vibrations was reduced to 4 μm, which completely satisfies the operational requirements. On passing through the critical frequencies the excess vibrational load did not exceed $15g$, which satisfies the requirements of strength and comfort while, when the ball bearings were rigidly fixed into the framework, the excess vibrational load at the critical frequencies of rotation reached a value of $120g$, which is not permissible

The compressor can be treated as an absolutely solid body over the whole range of frequencies. In order to do this, it is sufficient to select the pliability of the elastic mountings to be 5–10 times greater than the pliability of the doubly mounted rotor at its centre of mass, which is treated as a beam freely lying on two rigid mountings.

There is no need to introduce artificial dampers since they do not improve the rotor dynamics and reduce the efficiency. A reduction in the amplitudes and the excess vibrational load on passing through the critical frequencies is achieved by reducing the rigidity when installing the compressor into the elastic mountings.

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